FRAGMENTS

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Defining Continuity Mathematically and Philosophically

Frank Huang

This paper attempts to answer the following question: what is a continuum? In order to do this, I will lay out various historical explanations that attempt to describe the fundamental characteristics of a continuum. Continuity is a concept that has been discussed by philosophers ever since Ancient Greece. As such, there exists a multitude of conceptions of continua. The thesis of this essay states that while infinite divisibility is the most important condition of a continuum, it is not a sufficient condition on its own. After explaining infinite divisibility, I will survey other potential attributes and assess their applicability in terms of philosophical appeal and mathematical practice.

The notion of continuity ordinarily appears to us as an object or a phenomenon that is unbroken, or that has no gaps. Intuitively, one can think of certain mathematical objects such as lines, planes and solids as examples of continua; natural occurrences such as the ocean or the air surrounding us can also be thought of as different instances of this concept. The notion of continuity is one that is directly related to a multitude of other concepts in mathematics, such as the existence of points and the possibility of infinitesimals. As such, how continuity is defined has an impact on the nature and even the existence of these other concepts in mathematics. This makes continuity a significant concept in the philosophy of mathematics. The main debate surrounding continuity is the following: philosophically intuitive and appealing methods of explanation often are not in line with mathematical and scientific need and usage, and vice-versa. Albeit this problematic, members of the philosophical and mathematical communities have now reached some agreement concerning what the fundamental characteristics of a continuum consist of.

In this essay, I will attempt to answer the following question: what is a continuum? In order to accomplish this, I will list out and describe what philosophers until now have mostly agreed on in regard to the common features of various conceptions and instances of continua, as well as where each condition is situated in the spectrum of philosophical versus mathematical appeal. There are a total of six conditions that I will analyze in this essay: (1) infinite divisibility, (2a) punctilious points, (2b) intransigent infinitesimals, (2c) atomless gunk, (2d) intuitionistic indecomposable, and (2e) prodigious possibilities. The main claim of my essay will be the following: while infinite divisibility is the most important condition of a continuum, it is not a sufficient condition on its own. That is why there is a need to list out other possible explanations. After analyzing every one of these conditions, I will give an account of the applicability of each, and discriminate between the better ones and the lesser ones.

Before starting to describe the common characteristics of continua, it is relevant to put things in perspective by defining the notion of discreteness, which is basically the opposite of the concept of continuity. As opposed to something that is continuous, an object that is discrete is one that is separated[1]. Even if we amass a large sum of discrete objects, they will always remain separate unities, contrary to the parts of a continuum, which cannot exist independently outside of the continuum. It may be clearer if we use an example. We can think of a pile of pebbles as an instance of discreteness: while they are together and form a whole in the form of a pile, the latter can be decomposed, since every pebble is its own, separate unity. In contrast, if we think of the notion of time, there cannot be a single instance of time that exists out of the continuum of time: even if we categorize various parts of time using measures such as minutes, hours and days, we cannot cut out a single chunk of time in the same way we are able to single out a pebble from the pile. As Aristotle says: "discrete are number and language; continuous are lines, surfaces, bodies and also, besides these, time and place."[2] While something made out of the sum of discrete objects is only divisible[3] a certain number of times — i.e. until the whole is decomposed in such a way that we end up finding ourselves only with single instances of the initial discrete objects, which are indivisible—, a continuum is infinitely divisible. In addition, a continuum can also be divided anywhere. Reeder

1   Patrick Reeder, "Labyrinth of Continua." (Philosophia Mathematica 26, no. 1, 2017), 3.
2   Aristotle. The Complete Works of Aristotle: The Revised Oxford Translation. (Princeton, Princeton University Press, 1984), Categories VI, 4b20–25.
3   Reeder, "Labyrinth", 3.

sums this distinction in the following manner: "Discrete quantities exhibit distinctness and plurality; continuous quantities exhibit coherence and unity."[4]

As Reeder states, there is little to no opposition to the view that (1) infinite divisibility is a fundamental characteristic of all possible continua.[5] However, he notes that while infinite divisibility is a necessary condition of continuity, it is not a sufficient one. To illustrate this, he appeals to rational numbers and irrational numbers using Euclid's equilateral triangle. For the sake of simplicity, I will simply describe the issue using rational and irrational numbers. The problem is the following: while the rational plane satisfies the condition of infinite divisibility, since there can always be a lower decimal between two adjacent rational numbers, the existence of irrational numbers signifies that there are 'holes' or 'gaps' between certain rational numbers. In this sense, we may say that the plane consisting of rational numbers is indeed infinitely divisible, but we cannot infer from this that it qualifies as a continuum. Since we now know that infinite divisibility is not a sufficient condition for continuity, we must turn to and explore other possible conditions.

Reeder divides the following five conceptions into two categories: those that embrace a top-down view of continuity and those that see it from a bottom-up perspective. The top-down view takes the continuum as it is —i.e. as something that is continuous— and then attempts to analyse its parts. The bottom-up conception is the opposite: it sees the continuum as a sum of or as built up by its parts, the latter being the more fundamental elements of the continuum. Ancient philosophers such as Aristotle and Euclid functioned according to the top-down approach: both saw mathematical objects such as lines and planes as continua first, that could then be separated and cut into different sections or segments.[6] The top-down approach expresses the intuitive belief that continua exhibit the characteristics of wholeness, unity and coherence. As opposed to older philosophers, many more contemporary thinkers, including seventeenth- and eighteenth-century philosophers, have adopted the bottom-up approach. Indeed, philosophers and mathematicians such as Leibniz worked under the assumption of the existence of infinitesimals[7], which supposedly composed

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4   Ibid, 6.
5   Ibid, 3.
6   Ibid, 5.
7   Infinitesimals are extremely small quantities, of which the value infinitely approaches zero, but that are not equal to zero. Infinitesimals are so small that there exists no multiplier that can increase their value.

continua. This should be to no surprise, since Leibniz developed calculus, which fundamentally presupposes that curved lines are composed of indefinitely short straight lines. While a bottom-up approach worked well in a mathematical setting —e.g. it allowed the advancement of calculus, which is at the basis of many mathematical practices nowadays—, it does not fit our philosophical intuitions. This here is the root of the conflict between defining continuity for mathematical practice and defining it to satisfy philosophical appeal: top-down conceptions of continuity are compatible with our philosophical intuitions but are generally not appropriate for mathematical usage, while bottom-up approaches support mathematical practice but do not seem suited to our philosophical needs. It is thus relevant to explore conceptions deriving from both views, in order to have a more accurate and nuanced portrait of the supposed nature of continuity.

The next condition we are going to explore stems from the bottom-up approach: (2a) punctilious points. One of the major debates surrounding continuity is its relationship to points.[8] Euclid famously defined a point as "the ends of a line" and a line as that which "lies equally with respect to the points on itself."[9] While Euclid assumes this kind of relationship in postulating the nature of points and lines, many philosophers that have come after Euclid doubted this relation. For instance, Richard Dedekind and Georg Cantor, who have worked in the fields of calculus, analysis and in the creation of set theory, have incorporated the notion of points in the definition of continuity. The condition of punctilious points goes as follows: "an object X is continuous if and only if X is composed of a linear ordering of points and for any partition A, B of X such that all members of A are less than all members of B, there is exactly one point c that is either the greatest member of A or the least of B."[10] Dedekind and Cantor came up with this condition in order to define continuity without appealing to spatial intuition or geometric notions. Indeed, their unwillingness to make use of the latter two stems from mathematics shifting towards more logical and rigorous methods. While they have succeeded in the above-mentioned task, this may be at the cost of some explanatory power. A major objection against this description is the impossibility to verify that two lines, when oriented in a non-parallel manner, actually intersect. If two lines are not parallel to each other, it follows logically that their paths must meet at some point (no

8   Reeder, "Labyrinth", 7.
9   Euclid, "Elements, Book I." (Worcester, Clark University, Accessed in 2017).
10   Reeder, "Labyrinth", 8.

pun intended). However, if we define a line as a continuum, and a continuum as a collection of points adjacent to each other, one can come to the understanding that there exist gaps between these points. Indeed, since points "have no parts"[11] , the fact that they are adjacent implies that they are not immediately adjacent, which translates into the existence of spaces between points. If this is the case, how, then, can we make sure that the intersection of two lines is really a point, and not the empty space between two points? In other words, how can we be sure that there is a point of intersection? The fact that we cannot is an unavoidable weakness of punctilious points. Conceptualizing a continuum in this way "is to view the continuum as a hyper-plurality rather than a unity."[12] Its failure to satisfy our philosophical intuition that sees a continuum as a whole has us turning towards other possible explanations.

The next possible solution to our puzzle also follows the bottom-up tradition. As I have mentioned earlier, Leibniz is famous for 'inventing' calculus. Analogous to the practice of calculus is the assumption that infinitesimals exist. Indeed, calculus would not function without this presupposition. It comes as no surprise that Leibniz also defines continuity in terms of infinitesimals. His condition of (2b) intransigent infinitesimals goes as follows: "an object is continuous only if it is composed of infinitesimal magnitudes."[13] This assumption is what allows us to, for instance, find the area of a circle. Indeed, if we see a pentagon as the combination of five triangles —of which one of the sides are the sides of the pentagon and the opposing vertices being the middle point of the pentagon—, we can find the pentagon's overall area by adding the areas of the five triangles, which can be found using the inside angle and the height of every triangle. This formula works for all regular polygons; the number of triangles being the only datum that changes. For a square, there are four triangles, for an octagon, there are eight, and so on. A circle can be conceived as a polygon with uncountable sides which are infinitesimally short, of which the area is the addition of the areas of the uncountable triangles stemming from the uncountable sides. Using the radius of the circle —which is equivalent to the height of the inner triangles—, their inside angles and "some very simple algebraic manipulation, the proper result of $\pi r2$ follows."[14] We can see that Leibniz's view on continuity bears fruit in the mathematical realm. However, is it problematic in

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11   Euclid, "Elements."
12   Reeder, "Labyrinth" 9.
13   Ibid, 10.
14   Ibid, 11.

terms of philosophical considerations? It seems so. Leibniz's intransigent infinitesimals violates the Archimedean property, which stipulates that all numbers are, in some way, comparable. If formulated mathematically, the Archimedean property can be defined as follows: "for any values x and y where x < y, there is some positive integer n such that nx > y."[15] However, for infinitesimals, this is not the case. Infinitesimals are, by their nature and by their definition, infinitesimally small, and cannot be altered by integers. This being the case, it may be relevant for us to yet again move on to another possible solution.

As was stated earlier, Aristotle was one of the first philosophers to examine the notion of continuity. For Aristotle, a continuum is not composed of points. However, that is not to say that points do not exist: points are instead 'locations' or 'markers' situated on the said continuum. Stemming from the Aristotelian tradition is the conception of (2c) atomless gunk, which stipulates that "an object is continuous only if all of its parts have a proper part."[16] We can see that this is a top-down approach. Indeed, Aristotle mentions that "nothing that is continuous can be composed of indivisibles."[17] This explanation seems very appealing at first glance, since it relates well to our intuitive, infinitely divisible, ever-flowing conception of continuity. It complements our initial condition of infinite divisibility: this new condition 'fills the gaps' that infinite divisibility could not, so to speak. However, in regard to this new explanation, we can ask: what do we do without points as parts of the continua? As Frank Arntzenius argues, "electric fields, mass densities, gravitational potentials, etc. [...] are standardly represented as functions from points in space and time to point values. [Physics] would seem to make no sense if time and space did not have points as parts."[18] It seems that we cannot completely eliminate points from the notion of continuity, since science has progressed in such a way that points have become essential to scientific practice. Is the solution still elsewhere?

It comes as no surprise that intuitionists like Hermann Weyl and L.E.J Brouwer have also favoured the top-down approach to continuity. They have formulated the property named (2d) intuitionistic indecomposables, which states that "an object is continuous only if it is not identical to a sum of any of its disjoint proper parts."[19] Following this description, a continuum

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15   Ibid, 13.
16   Ibid, 14.
17   Aristotle, Physics, 231a23–25.
18   Reeder, "Labyrinth", 20.
19   Ibid, 21.

has one or more properties that are additional to the character-istics of the sum of its individual parts. As Weyl claims himself: "within a continuum, one can very well generate subcontinua by introducing boundaries; yet it is irrational to claim that the total continuum is made up of the boundaries and subcontinua. The point is, a genuine continuum is something connected in itself, and it cannot be divided into separate fragments; this conflicts with its nature."[20] By this, Weyl means to argue that while it is possible to divide a continuum into different parts by introduc-ing boundaries inside of it, this does give a particular onto-logical status to the various parts outside of the continuum. What Weyl names 'subcontinua' are fundamentally meant to exist as parts of larger continua, not on their own as individual instances. While this view is also philosophically engaging, there seems to be a fundamental issue in what it implies. How is an object not identical to the sum of all of its disjoint proper parts? We can illustrate this problem using an arithmetic analogy. Nor-mally, we would say that 1+1=2. However, positing that an ob-ject is not identical to a sum of its disjoint parts is analogous to positing that 1+1 does not equal 2. There seems to be a contra-diction here. In addition to this, the intuitionist approach is also subject to Arntzenius' objection regarding points that under-mined the Aristotelian model. While there exists some justifi-cation that have been spelled out by various mathematicians in reply to these two objections[21], these replies are not strong enough to properly refute the latter, and thus are not worth mentioning here. Let us move on to the last potential solution.

Charles Sanders Peirce, after exploring much of the above-men-tioned content about continuity, introduces a wholly new re-quirement, which Reeder labels as (2e) prodigious possibilities. According to this requirement, "an object X is continuous only if for any cardinal number κ and for any two distinct points on X, there are at least κ parts between those points."[22] For Peirce, a continuum is composed of parts, which are themselves com-posed of series. He also states that the collection of the parts of a continuum can be greater than any given multitude.[23] Peirce's motivation in defining his own conception of continu-ity is to be able to spell out all possible magnitudes using only one continuum. For Peirce, a continuum is divisible by all car-dinal numbers. As such, Peirce's conception of continuity can be expressed with surreal numbers, which are the combination

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20   Ibid, 23.
21   Ibid, 23-29.
22   Ibid, 29.
23   Ibid, 30.

of real numbers, infinite numbers and infinitesimal numbers. Indeed, since Peirce's continuum is divisible by all cardinal numbers, it can be properly expressed as the compilation of all numbers: the surreal numbers. Given their extent, one may ask how useful surreal numbers actually are. Reeder suggests that surreal numbers do not bring anything particularly new to the table, because mathematicians have yet to develop "a full analysis of the surreals", nor a "surreal integral".[24] This points to the possibility that Peirce's conception might shed light on the notion of continuity upon further analysis and investigation.

After surveying the main conceptions of continuity, it has come to my attention that they all have advantages and, more importantly, that they all have flaws. Despite this, the one conception that appears to me as the most promising is (2e) prodigious possibilities. Indeed, in contrast to the other four sub-conceptions that were introduced in this essay, the conception of prodigious possibilities does not seem to be facing any crippling objections, and it appears to be a great addition to the concept of infinite divisibility, since it is able to surmount the 'irrational numbers argument' that proved to be difficult for the latter. The condition of prodigious possibilities is both philosophically appealing, since it describes continua from a top-down approach, and mathematically practical, since it makes use of the compilation of all numbers, the surreal numbers. However, while this conception may be the most promising, it cannot be taken as flawless, as it is still the target of certain uncertainties, such as its relationship to points. As such, it may still be too early to single out one particular condition and claim it to be the best one. In this respect, it has become apparent that continuity is a concept that is still relatively ill-defined, and that still needs much philosophical consideration.

As was said in the introduction of this essay and as was illustrated in the subsequent paragraphs, continuity is a concept that is caught between philosophical appeal and mathematical practice. The explanations of continuity that are engaging philosophically and that satisfy our philosophical intuitions have issues in terms of mathematical and scientific applicability, while those that are useful in regard to mathematical usage appear to us as odd and illogical from a philosophical perspective. While the debate between mathematical usage and philosophical intuition is the main problematic in defining continuity, another more specific subject of inquiry concerning the nature of continua is its relation to points. Some of the theories we have

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24   Ibid, 32.

explored in this essay define continua as composed of points, while others argue that points exist outside of continua, or even that points do not exist at all. In hindsight, it may be that the concept of points ontologically precedes the concept of continuity. As such, further inquiry upon the nature of points and their existence inside the framework of continua could be relevant, in the scope of advancing our comprehension of these mathematical objects.

Aristotle. *The Complete Works of Aristotle: The Revised Oxford Translation.* Edited by Jonathan Barnes. Princeton, NJ: Princeton University Press. 1984.

Euclid. *"Elements, Book I."* Clark University. Accessed November 15, 2017. https://mathcs.clarku.edu/~djoyce/java/elements/bookl/bookl.html.

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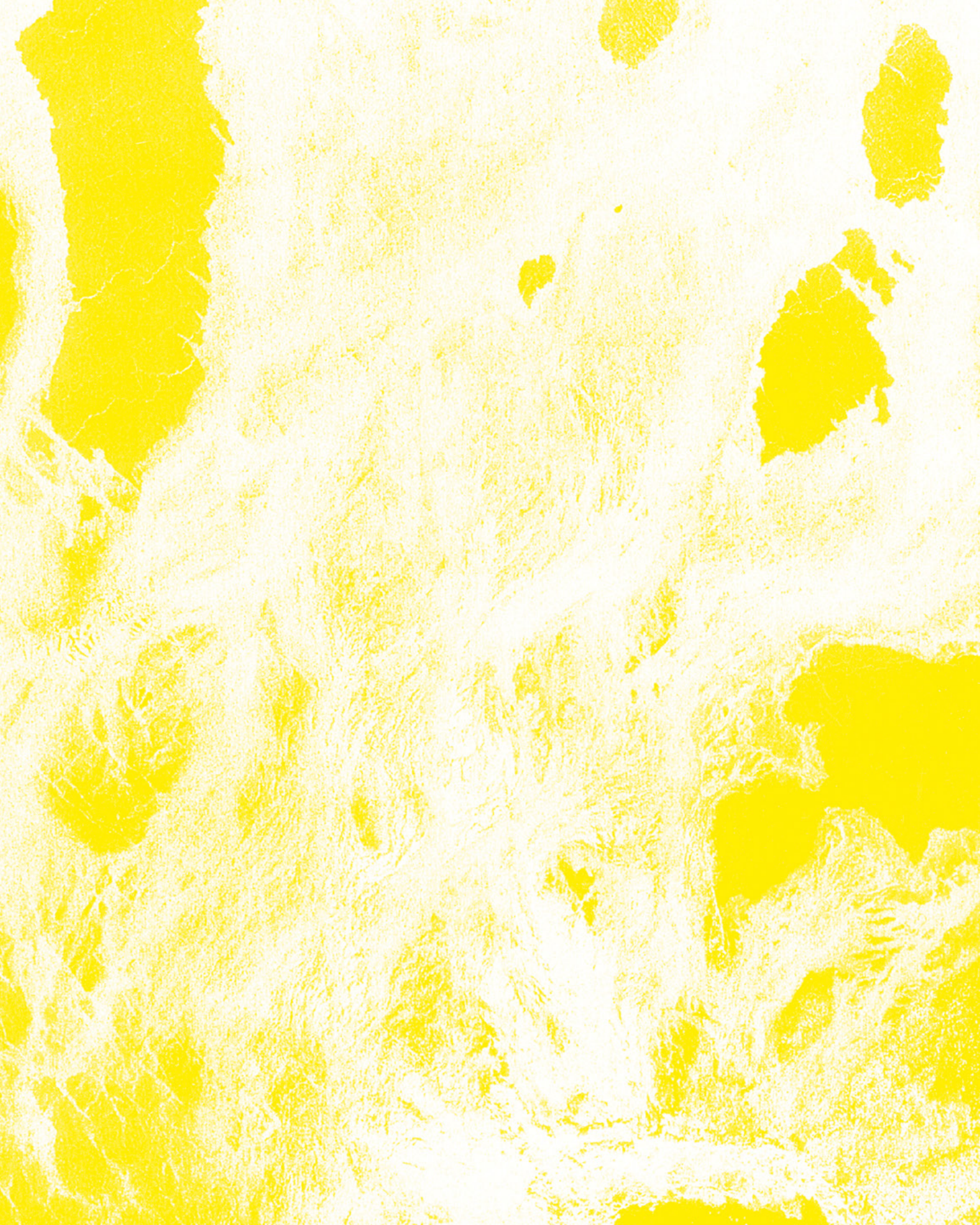
"Tasting Ain't Just In The Mouth!"

Jin Lee

This paper explores to what extent we can speak of food as being 'art.' Not everything is art — that much is clear. But it is practically relevant to determine whether the things we eat can fulfill artistic criteria similar to visual artworks. Philosophical aesthetics contends, after all, with questions about whether such things as pornography, animal creations, or avant-garde works can be called 'art.' Food presents complex theoretical issues along those lines; but it is also unique in that no other prospective art form can claim the sense of taste as its medium.

My analysis is preliminary, and should by no means be viewed as conclusive. The points raised are taken from Carolyn Korsmeyer and Elizabeth Telfer, two thinkers who take a tentative attitude towards edible artistry. We feast with our eyes on well-made painting – but a literal feast gives reason for hesitation. For can we, in fact, glean anything of artistic value with our mouths?

If one asks whether food constitutes art, one could claim either that a) food is food, while art is art, and we should not forcefully marry these two concepts together; that b) food is artwork in a minor sense of 'art,' such that culinary art is limited in ways that conventional art forms (like painting, sculpture, or music composition) are not; or that c) food is as much an art form as these, in that culinary stuffs can fulfill all of the relevant criteria which other art forms can. My aim is, then, to reject a) and b) and to defend claim c). I do not wish to understate that food presents an unusual case for art theorists; but I claim food can, as much as paint on canvas, beget artistry.

First, as per a), there are some justifications for the claim that food and art are best left to their respective domains. For one, it is proposed by some that in order for something to be 'art' it must be considered for no sake other than its own enjoyment. This argument maintains that food is biologically necessary, and thus has a functional value that is at odds with artistic dis-

interest.[1] But it is not always the case that food serves a rote biological function – we often appreciate food along multiple dimensions (e.g. taste and gustatory appeal),[2] and not merely for nourishment. If food were just consumed for biological reasons, there would be no use in discussing different tastes, flavours, and smells. But there exist institutions like those of the gourmet and food critic, focused on the tastes and scents of foods, which dispels the notion that food is just nutritious. As one appraises a painting for qualities like colour and depth, one can also appraise dishes for their sensation or balance upon the palate.

A second argument against food as art might follow, however, by questioning if tastes or gustatory sensations really merit aesthetic value. One might say, for example, that i) a source of enjoyment is lesser if it depends solely upon bodily (rather than intellectual) stimulation, which seems to apply to food, and that ii) taste offers no substantive insight into anything other than what tastes agreeable. As William Deresiewicz declares, food is not art because while both art and food begin at the senses, sensual pleasure is where food stops doing work.[3] It seems food offers rote pleasure, but it is not itself representative[4] of more than just that. An analogue we could also call a 'lowbrow' source of pleasure is, for example, sexual intercourse. Eating food and having sex are both bodily functions borne from necessity, but both are incidentally capable of bringing pleasure. Sexual urges can be satisfied to varying degrees, though the desired result is always the same, just as how one's appreciation of good food can vary in degree. However, the pleasurable feeling one receives from good food – as with sex – just seems to be that it was good.

In response to the above, however, we can reject a questionable assumption. To say that nothing much to proclaim about food, other than that it tastes good or bad, is to presume that the gustatory senses have nothing to do with higher cognizance. One might say, in other words, that we cannot coax profound reactions from our taste buds due to their limited nature. This is untrue; for while our gustatory and olfactory senses may be habitually underdeveloped[5] in lieu of visual and audial emphasis,

1   Telfer, Elizabeth. "Food as Art." Arguing About Art, Third Edition: Contemporary Philosophical Debates. Eds. A. Neill & A. Ridley. Routledge: Abingdon, United Kingdom. Published 2008. Pages 11-29.
2   Ibid.
3   Deresiewicz, William. "A Matter of Taste?" The New York Times. Published 2012. Accessed Nov. 21, 2017. URL < http://www.nytimes.com/2012/10/28/opinion/sunday/how-food-replaced-art-as-high-culture.html>
4   Ibid.
5   Telfer, Elizabeth. "Food as Art." Arguing About Art, Third Edition: Contemporary Philosophical Debates. Eds. A. Neill & A. Ridley. Routledge: Abingdon,

that is no reason to say we cannot refine our sense of taste nor give it more depth. So the analogy between food and sex seems inaccurate; it might be true that the pleasure drawn from sexual fulfillment is one-dimensional, but this does not have to be the case for taste.

Now I will switch tactics from contending with a) to contending with b). Recall that b) was the claim that although food may have limited aesthetic value, it cannot fulfil some criteria other arts can fulfil. To reject this, we can point to instances whereby food leads to as significant or as reflective an experience as that gained from conventional art. Consider, for instance, in the film Ratatouille, how food critic Anton Ego remains uninspired until he tastes the protagonist's rendition of the titular dish. Ego's reaction is revelatory: Not only is it as a ratatouille should be, but it transports him away to a nostalgic reverie for his mother's cooking and also rekindles his affection for French cuisine. What is more, the dish is also expressive: The simplicity and purity with which it is constructed causes the villain to appreciate, as he never did before, the powerful idea that anyone can cook well if they give their best effort. What else should one call this sort of reaction, if not something akin to an art critic's reaction to a masterpiece?

Marcel Duchamp's Fountain. Replica. Image from Wikimedia Commons.

Food can take on meaning or contain representative value — that much seems true. But an opponent may still argue that the meaningfulness food takes on is, itself, a limited one. When food means something to us, it is often imbued with either cultural,

religious, or commemorative importance.[6] Artworks, however, are to be valued in an autonomous[7] manner, so that they are appraised without reference to external facts about them. Thus the problem is not that food has no meaning, but rather that it only has context-dependent meaning. Artwork does not suffer from this limitation, if Korsmeyer is to be believed, since by their nature artworks are abstracted from surrounding contexts.[8]

The above argument fails to consider whether the characteristic it flaunts as being unique to art — context-free meaningfulness — actually applies to a majority of cases in fine art. Let us say that what we mean by 'fine art' are works usually displayed or shown to us. If art is meant to be presented, is it not the case that the public context through which art is displayed plays a vital role in its assessment? If so, art critics and audiences also seem to fail to abstract their enjoyment of works from external contexts. Duchamp's work Fountain is, after all, a toilet. The point is that it is exhibited to us as an artwork.

Conversely, if we took a beautiful painting that should be exhibited in a public context, but instead hung it without fanfare in a private dining room, it is as likely to be seen as furnishing or to be ignored as it is to be appraised. So if part of what makes something 'art' is its context, what is it that is so objectionable about food?

I have shown that there are many arguments against the claim that 'food is art,' and that they all fail to carry their weight. We first considered whether art and food are incommensurable, if only because food is necessary and art is not. Next, we considered whether food is a limited art form because its reliance on bodily pleasure makes it lesser. Finally, we rejected the view that art belies only a context-free value that precludes the cultural, traditional, or personal meanings that food can take. So if I have been convincing, we should claim that there is no reason to insist food cannot be art.

6   Korsmeyer, Carolyn. "The Meaning of Taste and The Taste of Meaning." Arguing About Art, Third Edition: Contemporary Philosophical Debates. Eds. A. Neill & A. Ridley. Routledge: Abingdon, United Kingdom. Published 2008. Pages 30-49.
7   Ibid.
8   Ibid.

Telfer, Elizabeth. *"Food as Art."* Arguing About Art, Third Edition: Contemporary Philosophical Debates. Eds. A. Neill & A. Ridley. Routledge: Abingdon, United Kingdom. Published 2008. 11-29. Print.

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Duchamp, Marcel. *Fountain.* Photo taken 24 July 2005 by spDuchamp. Wikimedia Commons. Web. https://commons.wikimedia.org/wiki/File:Duchamp_Fountain_(28365079).jpg

Culture and alienation in Hegel's *Phenomenology of Spirit*

Stefan Davies

The goal of this essay is to give an account of how the alienated self creates the world of culture. This essay is primarily an exposition of Hegel's discussion of culture and self-alienated spirit in segment 6 of his Phenomenology of Spirit. What I try to elucidate in this essay is the dialectical movement of self-consciousness as it tries to give up its own individuality in favour of the universality of the state. Both the state and wealth play central roles in this dialectic and at different times, the individual in this world views the state to be the good and wealth to be the bad and vice versa. As is characteristic of Hegel's dialectic, these distinctions turn out to be their opposites. Ultimately, it is language that mediates the alienation of the self. This alienation is manifested in society, the result of which is the destruction of the previous political order and the creation of a new one.

Alienation is the "mediation," for the "cultivation for the self."[1] It is through culture that the individual "acquires standing and actuality," and this is achieved through the self's alienation.[2] In the first part of this essay, I will explore what Hegel means by this notion and the separation of self-alienated spirit into "actual consciousness" and "pure consciousness."[3] Then, I will discuss how this notion of alienation is manifested in culture. In particular, I will explore the development of what Hegel calls the "noble" consciousness[4]. It will then be important to discuss the role of language in the alienation of the noble consciousness and the transition from Feudalism to Monarchy. At the conclusion of this essay I will explain why for Hegel, culture is the product of alienation.

To introduce this notion of alienation, I think it is useful to consider Jean Hyppolite's discussion of the difference between the "pedagogy" of the Enlightenment and that of humanism, in

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1   Jean Hyppolite. Genesis and Structure of Hegel's 'Phenomenology of Spirit', trans. Samuel Cherniak & John Heckman (Illinois: Northwestern University Press, 1974), 384.
2   G.W.F. Hegel. Phenomenology of Spirit, trans. A.V. Miller (Oxford: Oxford University Press, 1977), 298.
3   Hegel, 302.
4   Ibid, 305.

comparison to Hegel's own conception of the development of the self.[5] In the Enlightenment conception, development of the self or individual is a linear progression forward and in humanist pedagogy, there is a "spontaneous and harmonious development of all the forces of nature."[6] For Hegel, in education there comes a moment in which the self "becomes unequal to itself and negates itself, thereby gaining its universality."[7] This moment is that of alienation. It is clear then, that for Hegel, this self-negation or opposition with ones' self is characteristic and necessary for the development of the self. In the world of culture, the individual will gain standing only when he becomes alienated from his "natural" being.[8] According to Hegel, any "presumed individuality" is only "imaginary" and is out of place in a world in which only that which has "externalized itself, and therefore only the universal" will obtain an "actual existence."[9] It appears that in order for an individual to find his place in this "alienated world" which has the form of a "fixed and solid reality" he must occupy himself in "conforming," to it.[10] In this sense, the individual in this society is confronted by a world of law and custom and the only way to become a part of that world is to negate himself to move towards what is universal. The individuality of the self must be negated in order to become a part of the world of culture and in doing so, the self will become alienated from its natural being. How this alienation manifests and creates the world of culture will be the next task of this essay. Before that, however, it will be necessary to clarify the separation of self-alienated spirit.

Self-alienated spirit consists of both pure consciousness and actual consciousness. The former is "the unity of the self and essence," whereas the latter is "the consciousness of an objective real world freely existing on its own account."[11] Pure consciousness is a moment of actual consciousness and considers itself to be apart from the latter and takes the twofold form of faith and pure insight.[12] These two determinations, however, are dealt with at the end of this section but I will not discuss them in this essay. In paragraph 493, Hegel distinguishes between the "thoughts of Good and Bad" which are "represented within

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5   Hyppolite, 305.
6   Ibid, 385.
7   Ibid, 385.
8   Hegel, 295.
9   Ibid, 298.
10   Ibid, 299.
11   Ibid, 295.
12   Stephen Houlgate, Hegel's 'Phenomenology of Spirit': A Reader's Guide (London: Bloomsbury Academic, 2013), Self-alienated spirit.

self-consciousness."[13] The Good is taken to be "the self-accordant, immediate, and unchangeable essence of every consciousness" which is the "independent spiritual power of the in-itself."[14] On the other hand, the Bad is "the essence that is null and invalid."[15] The main difference here, is that whereas the Good is taken to be that in which individuals find what is universal, the Bad is what perpetually leads to a "return-to-self."[16] These abstract thoughts of Good and Bad are objectified by actual consciousness as "state power," and "wealth."[17] State power is the "universal 'work'" in which individuals find their "essential nature expressed."[18] In this way, work is something that is contributed to by all and so it represents the universal foundation for all individuals. It is something that every individual is involved with and because this work is for everyone, individuals operate as "being-for-another."[19] Rather than being purely self-interested, individuals here work for their fellow man. Its opposite, wealth, is "passive" and "devoid of inner worth" and yet wealth is something that is produced by the labour of all and subsequently, the fruits of this labour is also to be enjoyed by all.[20] Interestingly, whilst each individual has in mind their own self-interest, or rather take themselves to be self-interested, they cannot help but work in such a way that others benefit. What Hegel has in mind here is the notion of the free-market economy that was championed by Adam Smith, in which the self-interest of each individual leads to the benefit of everyone since in order to gain wealth, one must produce things for other people.[21] Thus, the "being for himself" of an individual in this society is actually universal.[22]

Pure consciousness considers itself separate from the objective distinctions of state power and wealth. It believes it can choose to identify with either of these objectifications, or neither of them. Self-consciousness is the "relation of its pure consciousness to its actual consciousness, of what is in the form of thought to what exists objectively."[23] In this sense, it is "judgment."[24] As such, the immediate determination of the "two sides

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13 Hegel, 301.
14 Ibid, 301.
15 Ibid, 301.
16 Ibid, 301.
17 Ibid, 301.
18 Ibid, 301.
19 Ibid, 301.
20 Ibid, 301.
21 Hyppolite, 394.
22 Hegel, 302.
23 Hegel, 302.
24 Ibid, 303.

of objective reality" is that state power is the good and wealth is the bad since the former is taken to be the "in-itself" and the latter as "being-for-itself."[25] However, Hegel notes that as "spiritual moments," each is the "interfusion of both moments" and since self-consciousness is both in and for itself, it must be related to state power and wealth in a "twofold manner," with the result that their nature as "self-alienated determinations" will be revealed.[26] Self-consciousness holds an object to be good and to "possess intrinsic being" in which it "finds itself" and that to be bad in which it finds "the opposite of itself."[27] In addition, what it takes to be good and bad, it takes to be "intrinsically good and bad."[28] Self-consciousness finds in the state power its "simple essence and substance in general," however, it does not find its "individuality."[29] Instead, it finds that the state "subdues" individual action into "obedience."[30] This is to say, self-consciousness, a being that is in and for itself, finds intrinsic being in state power but at the same time it finds a suppression of individuality and so as a judge presiding over these objectifications, it takes state power in this case to be "the Bad."[31] On the other hand, it takes the Good to be wealth, which leads to the "general enjoyment" of all.[32] Hegel notes that even if wealth does not satisfy everyone at all times, this does not detract from its "universal and necessary nature" by which it imparts itself to all and acts as a "universal provider."[33] Despite these initial judgements, self-consciousness comes to realize that within the state, which constitutes and promulgates the law and government, the individual finds "his ground and essence, expressed, organized and manifested."[34] Moreover, the individuals' experience of wealth is ultimately unfulfilling since he only experiences enjoyment of himself "qua single and independent individual."[35] What is lacking here is the experience of the individual's "universal nature."[36] As such, the "Notions of Good and Bad" are the opposite of what they were initially taken to be.[37]

In paragraph 499, Hegel notes that actual consciousness has "within it both principles," that is the Good and the Bad, and the

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25   Ibid, 303.
26   Ibid, 303.
27   Ibid, 302.
28   Ibid, 302.
29   Ibid, 303.
30   Ibid, 303.
31   Ibid, 303.
32   Ibid, 303.
33   Ibid, 303.
34   Ibid, 304.
35   Ibid, 304.
36   Ibid, 304.
37   Ibid, 304.

distinction between the two "falls solely within its own essence, viz. in the relation of itself to the actual."[38] Indeed, in this segment, Hegel is not saying that state power is good and wealth is bad or vice versa, or indeed that consciousness takes this to be the case. Instead, consciousness finds the Good and the Bad in both objectifications but these judgements are reversed in the course of this dialectic. Hegel introduces the distinction between the noble consciousness and the ignoble consciousness. The former is that which finds both state power and wealth of "like nature to itself."[39] In state power, noble consciousness sees in public authority "its own simple essence" and in the service of that authority "its attitude towards it is one of actual obedience and respect."[40] With regards to wealth, noble consciousness sees that it "procures for it awareness of its other essential side," which is the consciousness of "being for itself."[41] The ignoble consciousness, however, "clings to the disparity between the two essentialities," that is being-for-self and being-in-itself, and so it takes state power to be suppressing its own being-for-self and therefore this individual "hates the ruler" and is always "on the point of revolt."[42] In wealth, it sees only the enjoyment of "its own self-centred existence" and the "disparity with its permanent essence."[43] Hegel then proceeds to focus on the development of the noble consciousness, which, being so "positively related" to state power, begins to negate its own being in order to serve the state.[44] This is the "heroism of service, the virtue which sacrifices the single individual to the universal."[45] In this way, through the alienation of its natural being and individuality, noble consciousness develops into an "essential existence." and the state too, is actualized by this alienation.[46] As a result of the sacrifice of its own individuality, noble consciousness "wins self-respect and the respect of others."[47]

Despite this sacrifice, there remains a problem for both the individual and state power. State power has no "particular will" yet and so it is not in a position to decide amongst the different opinions offered as "counsel."[48] In addition, self-conscious-

38   Ibid, 304.
39   Ibid, 304.
40   Ibid, 305.
41   Ibid, 305.
42   Ibid, 305.
43   Ibid, 305.
44   Ibid, 306.
45   Ibid, 306.
46   Ibid, 305.
47   Ibid, 306.
48   Ibid, 307.

ness has not yet "renounced its own pure self."[49] The ultimate way for the individual to negate himself and give up his own being in the name of the state is "death," however, since the individual remains alive, it is not clear to others that he is truly committed to the "general good," and so his counsel is "open to suspicion."[50] His conduct therefore, appears to resemble that of ignoble consciousness. In this way, the ignoble consciousness is revealed to be the truth of the noble consciousness. It must also be noted that death, whilst it would of course demonstrate the utmost commitment of the individual to serve the state and to become universal, it would not provide the state with the will that it is lacking. The reason being, is that if the nobles who serve the state were to die, then there wouldn't be anyone left to recognize the state's power. This is much like the master who chooses not to kill the slave because he needs someone to recognize his power. Currently, the state only has power by virtue of the recognition that it is accorded from the nobles. The alienation, however, that allows noble consciousness to fully renounce its own being, whilst staying alive, "takes place solely in language."[51] In Hegel's discussion of sense-certainty, we saw that when natural consciousness tried to ground the knowing of an object in an I, this I, it expressed what is actually most universal, since it doesn't express anything specific because when one uses the word I, it could refer to any I. I might see an object over here, but another I might see a different object.[52] The important thing to note, is that this kind of language allows the individual, whether intentionally or not, to sacrifice what is particular to itself, in the name of the universal. Language allows the individual to become the universal. As Hegel puts it, "The 'I' is this particular 'I' – but equally the universal 'I.'"[53] As Houlgate notes, since the noble had previously failed to become one with what is universal since his continuing existence left his intentions open to suspicion, "his consciousness of his unity with the world must now find expression in a reality that is itself distinct from his own divided reality."[54] As such, language is the mediating factor in the alienation of the noble consciousness.

As this mediating factor, however, language has not completely united the two "extremes" of noble consciousness and state power.[55] What consciousness is still lacking is the "actual trans-

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49   Ibid, 307.
50   Ibid, 307.
51   Ibid, 308.
52   Ibid, 61.
53   Ibid, 308.
54   Houlgate, Self-alienated spirit.
55   Hegel, 309.

ference to it of the state power" and what is lacking in state power is that "it should be obeyed" not just as the general good but "as will."[56] This is to say that state power is still not actualized, that is, it is not embodied in a human figure. This comes about, however, when the noble consciousness adopts the language of "flattery."[57] As such, the "heroism of silent service" becomes the "heroism of flattery."[58] This marks the transition from Feudalism to the actualization of state power in an "unlimited monarch."[59] Noble consciousness "divests itself" of the "pure intrinsic being of its thinking, its very 'I'.[60] This power is actualized because the nobles give this monarch "his own proper name" and so now, the knowledge of the difference between this individual and everyone else is not simply "presumed" but it is "made actual by all."[61] In this way, the noble fully alienates himself and his own individuality and state power is no longer something abstract, it is embodied in the figure of the monarch that wields unlimited power. Thus the "language of their praise", the fawning and obsequious flattery of the nobles, is the spirit that "in the state power itself unites the two extremes."[62] This monarch, however, experiences an "independence that is self-alienated."[63] The problem is that his status results from the "nourishment from the sacrifice of action and thought by the noble consciousness."[64] Thus, the monarch does not experience unmediated power, instead, his power is nothing without the constant praise and sacrifice of individuality of noble consciousness. As a result, it is alienated. On the other hand, this noble consciousness "receives back the other extreme," this extreme being the power of the state in the form of wealth.[65] The noble, who previously viewed the state as something standing over and above him, now sees the state as a "source of wealth and satisfaction."[66] Initially, wealth is not "ensouled," but noble consciousness comes to realize that this wealth comes to have a reality of its own.[67] The noble consciousness feels itself to be controlled by market forces and is in "the power of an alien will."[68] In being satisfied, one is not satisfied because this satisfaction comes at the price of having to give up one's independence to

56   Ibid, 310.
57   Ibid, 310.
58   Ibid, 310.
59   Ibid, 310.
60   Ibid, 311.
61   Ibid, 311.
62   Ibid, 311.
63   Ibid, 311.
64   Ibid, 311.
65   Ibid, 311.
66   Houlgate, Self-alienated spirit.
67   Hegel, 313.
68   Ibid, 313.

those forces. As such, the individual feels totally alienated from himself and is completely given over to satisfaction. In this state of "disruption," the individual finds his personality is "dependent on the contingent personality of another."[69] That is, not only does the individual feel that he is subject to external forces of the market but he feels, in this sense of alienation, that his very self depends on others to the extent that he loses any sense of identity and individuality.[70] The individual, as a result of this feeling of alienation, becomes rebellious and comes to resent wealth. In this way, the distinction between noble and ignoble consciousness falls away and "both are the same."[71]

The individual is torn apart because he feels that his sense of self has been lost to this world, however, this is only because "he knows that he has a self to lose."[72] As such, this self-consciousness is "absolutely self-identical in its disruption."[73] This is to say, the individual is "conscious, in its outrage, of being internally inconsistent and confused."[74] In this way, this self-consciousness comes to realize that there is something wrong with wealth and more significantly, culture itself. The individual realizes that neither the thoughts of Good and Bad nor their objective actualizations of state power and wealth hold the "truth."[75] The truth of culture turns out to be that all these moments "become inverted, changing into one another."[76] The language of "this disrupted consciousness" comes to be the "perfect language" of the entire world of culture.[77] That is, this individual who feels this overwhelming sense of alienation, expresses outrage in feeling his loss of self and yet at the same time holds onto his sense of self.[78] This language becomes the perfect expression of what is a disrupted and alienated culture.

I could at this stage give more examples of how alienation is manifested in culture, and indeed this notion does appear in Hegel's discussion of faith and pure insight, but I believe that I have given sufficient examples of how it manifests in society. The task now is to explain why culture, for Hegel, is the product of alienation. I have touched upon this question implicitly throughout the course of this essay but I will now address it

69   Ibid, 313.
70   Houlgate, Self-alienated spirit.
71   Hegel, 314.
72   Houlgate, Self-alienated spirit.
73   Hegel, 316.
74   Houlgate, Self-alienated spirit.
75   Hegel, 316.
76   Ibid, 316.
77   Ibid, 316.
78   Houlgate, Self-alienated spirit.

explicitly. As noted at the beginning of this essay, alienation is necessary for the development of the self. The alienation of the individuals' natural being is crucial not just for the development of the self, but it also has important consequences for culture as a whole. Indeed, this self-negation gives rise to the unlimited power of the monarch. In this sense, the effect of alienation is the dissolution of the previous social or political order. This dissolution also occurs when the monarch, who, in realizing that his power is the result of the flattery of the nobles, finds himself to be in an alienated state. This alienation, then, ultimately results in the collapse of the monarchy. Importantly, it was self-alienated individuals who created this world of culture and they in turn brought about the collapse of these distinctions in society. These self-alienated individuals, then, come to have a more profound understanding of society and indeed human existence through their experiences. Self-consciousness, as a being that is for-itself and in-itself, tried to identify with the universality of the state as well as the for-itself of wealth, and in each iteration of this dialectical movement, it came to realize that the Good is neither to be found in the state nor wealth. Noble consciousness finds the Good and the Bad in both objectifications and yet through its experience it comes to realize that each is the "opposite of itself."[79] Indeed, ignoble consciousness comes to be the truth of noble consciousness and vice versa. Indeed, this is what it means to be alienated. One cannot be noble without being ignoble and one cannot be ignoble without being noble. The distinctions in society, that is the manifestations and actualizations of the Good and the Bad in society, dissolve away. Moreover, alienation has a creative aspect to it which results in the collapse of these distinctions and leads to a better understanding of the self and society from the perspective of self-consciousness. This dissolution results from the creative power of reason.

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79   Hegel, 316.
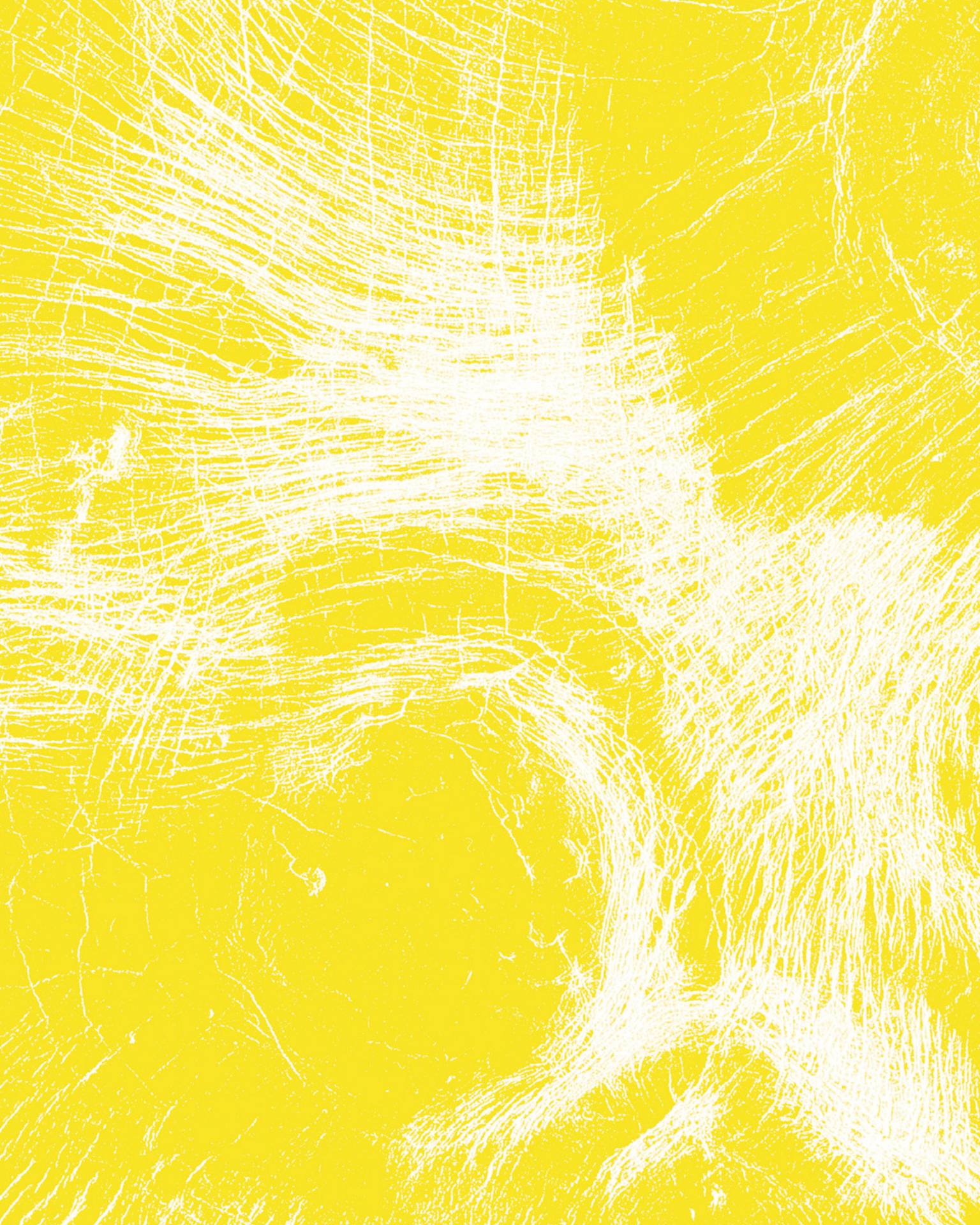
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The Idea for which Kierkegaard Can Live and Die

Lien Shi

Søren Kierkegaard introduces subjectivity as constitutive of a type of truth which traditional philosophy, particularly systematic rationalist accounts of existence, is unable to access and tends to distort, if not disregard entirely. For Kierkegaard, the existence of the individual is only capable of being understood in subjectivity. This paper is a close examination of a passage from Kierkegaard's Concluding Unscientific Postscript to Philosophical Fragments. I pose and answer four questions which deal directly with the claims in the passage. I begin by showing the relation between subjectivity, uncertainty, and decision. Then I explain the context in which objectivity can be erroneous. After, I specify how passion is the determinant of subjectivity. Finally, I show how subjectivity can constitute truth and conclude that truth is sincerely meaningful to the individual when it is pursued subjectively, rather than objectively.

In Concluding Unscientific Postscript to Philosophical Fragments, Søren Kierkegaard describes truth as being constituted by passion and subjectivity. His work is often misread by post-positivists as claiming that truth is purely relativistic and belonging to the individual, but this is not the case. Kierkegaard does not deny the certainty of objective truth, but maintains that what can be known through an objective approach to truth is limited. In the following passage, Kierkegaard makes several claims that distinguish the subjective and objective approaches to truth from each other:

> Only in subjectivity is there decision, to seek objectivity is to be in error. It is the passion of the infinite that is the decisive factor and not its content, for its content is precisely itself. In this manner subjectivity and the subjective 'how' constitute the truth.[1]

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1   Søren Kierkegaard, A Kierkegaard Anthology, ed. Robert Walter Bretall (Princeton, New Jersey: Princeton University Press, 1946.), 214.

I will explicate the passage by raising and answering four guiding questions to clarify how Kierkegaard argues that truth is only significant to the individual when it is pursued in subjectivity.

First, why is there decision in subjectivity, and only in subjectivity? If what Kierkegaard says about subjectivity is true, it logically follows that there is no decision in objectivity. The crucial difference between truth constituted in subjectivity and objective truth is the relation that the individual has to the truth. Objective standards assign truth values "not caring" how the individual recognises them, because in objectivity the decision is made for, not by, the individual.[2] Upon accepting the premises 'A is B' and 'B is C,' the individual is forced by objectivity to accept the conclusion that 'A is C.' The individual only knows with certainty that 'A is C' because it is an objective truth. Ultimately, objective truths are not meaningful to the individual because the objective truth values are decided outside of the individual. Rather than determining 'what' defines reality or existence, in subjectivity the individual is positioned to make a decision where there are no objective truth values assigned and no objective criteria that ever will decide on them, where the individual "merely has, objectively, the uncertainty."[3] Subjectivity is a question of 'how' the individual relates to the objective uncertainty, of whether or not the individual passionately appropriates the objective uncertainty into their life by inwardly choosing to believe it is true while acknowledging its uncertainty. When an objective uncertainty is appropriated in such a manner, one commits oneself to it with what Kierkegaard calls one's own "passionate inwardness."[4] The fact that the individual can only be confronted with a decision when there is uncertainty is precisely why there is only decision in subjectivity.

Second, why is seeking objectivity "to be in error"? In dissuading the attempt to be objective, Kierkegaard does not mean that objective truths are untrue, but that there is a limitation to how objective about truth any individual subject can be. For Kierkegaard, to exist as a subject is to separate one's self from objects, which means the self is only experienced in subjectivity.[5] To seek objectivity about matters related to the self, such as existence, is to seek a point of view which is outside of existence. The point of view outside of existence through which existence can be objectively systematized is solely accessible to God, "who

2   Kierkegaard, A Kierkegaard Anthology, 5.
3   Ibid., 214.
4   Ibid., 214.
5   Ibid., 205.

is outside existence and yet in existence."[6] Any attempt to use God's point of view, or an external point of view, to seek objective truth is misguided since one can only ever approximate, not appropriate, an external point of view. Seeking objective truth about objective uncertainties is erroneous; the individual that approximates truth forgets the limitations of their own existence and will fail to obtain the truth.

Third, why is passion, not content, the "decisive factor" of subjectivity? The answer lies in faith, "the highest passion in the sphere of human subjectivity."[7] To explain what Kierkegaard means by faith, I will state what he means when he writes "The idea of philosophy is mediation—Christianity's is the paradox."[8] [9] A paradox is not just uncertain but irreconcilable. To appropriate a paradox, to be capable of holding notions which are fundamentally irreconcilable, is to be intensely passionate. One cannot be passionate about something without objective uncertainty or risk, for that which is certain is objectively decided. Acknowledging the impossibility of reconciliation heightens the tension between the individual and paradoxical faith; accepting and embracing the absurdity of one's own faith, all while believing in it, requires the most intense passion. For Kierkegaard, the paradox need not be Christianity. One can claim to be Christian but without true faith, and be praying in "false spirit," while a Pagan can pray to an idol with "the entire passion of the infinite" and ultimately reach the "true God."[10] Faith, the highest passion in subjectivity, is rooted in the relation between the believer and the belief, not what the content of belief is, nor in the objective truth value of the belief. An objective approach to one's faith, where the object of faith is an objective uncertainty, is "thoughtlessness."[11] When truth is approached in subjectivity, it is constituted by the individual's relation to the uncertainty, not the recognition of a truth value. The closer to paradox the objective uncertainty is, the more passionate the inwardness that is required to truly believe, appropriate, and commit oneself to it as the truth.

Finally, how does subjectivity and the subjective 'how' constitute the truth? The truth which Kierkegaard is concerned with

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6   Ibid., 201.
7   Ibid., 210.
8   Ibid., 14.
9   Much of the canonical Christian Bible is paradoxical. For instance, the Virgin Mary is a mother. Her child, Jesus Christ, is human and mortal, yet divine and immortal.
10   Ibid., 212.
11   Ibid., 207.

is that "which is true for me, to find the idea for which I can live and die."[12] Objectivity or the objective 'what' gets one nowhere. If God can be understood objectively, that is, if God is to be understood outside of a subjective relation to him, God will mean nothing to the individual because one will not be related to him any more than if his existence is denied. If something which is an objective uncertainty is systematized and objectified by the individual subject, truth is approximated but never gained. Where objective truth cannot decide, the individual has a decision to make. The subjective 'how,' the generation of the passionate inwardness required to sustain a faithful belief in an absolute paradox, is what constitutes the truth which Kierkegaard wants, a truth which is incorporated in his being, and thus is something for which he can live and die.

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12   Ibid., 5.

Kierkegaard, Søren. A Kierkegaard Anthology.
ed. Robert Walter Bretall. Princeton, New
Jersey: Princeton University Press, 1946.

Merleau-Ponty's "Cézanne's Doubt": On the Endless Phenomenological Task of Painting

Natasha Beaudin Pearson

In this essay I offer an exegetical account of Maurice Merleau-Ponty's discussion, in his paper "Cézanne's Doubt," of the artworks of the painter Paul Cézanne. Specifically, I outline Merleau-Ponty's argument that Cézanne's painting, which marked a radical departure from the geometric perspective favored by Western art historical tradition up to that point, resembles the natural world insofar as it accurately expresses how we perceive it. First, I explain Merleau-Ponty's contention that Cézanne's artistic process, which sought to recreate and not merely imitate objects in nature – an impossible metaphysical task – constitutes a phenomenological approach, and this makes his artistic project also a fundamentally philosophical one. Next, I touch on the paradox Merleau-Ponty calls "the phenomenon of expression," namely the fact that artists seek to crystallize the lived experience(s) of a multitude of people by relying solely on their own personal experience. I conclude by presenting Merleau-Ponty's assertion that these issues have never stopped us – and will never stop us – from creating and engaging with art.

Throughout the history of Western art, paintings that abided most faithfully to geometric perspective tended to be lauded as the closest representations of the real world. Raphael's School of Athens is a prime example: all the figures and architectural structures recede toward and are scaled according to an invisible vanishing point, located at the exact center of the composition. The result is a picture of impeccable order, stillness and congruity. This, however, does not accurately reflect our lived experience of the world, claims Merleau-Ponty. In reality, our

senses perceive the world chaotically, distortedly, "in a shifting way:" we constantly experience illusions, such as "when we move our heads [we believe] that objects themselves are moving," before our reason "set[s] these appearances straight."[1] Thus, the painter that intends to truly capture the world that we see with our eyes must translate this "primordial," "spontaneous," "pure" nature by pictorial means, and this is no easy task.[2] However, Merleau-Ponty contends, one Post-Impressionist painter nearly succeeds: Paul Cézanne. I will explain Merleau-Ponty's assertion that Cézanne's paintings "penetrate right to the root of things," namely that they resemble the world that we see with our eyes; nevertheless, I will present reasons why a painting can never completely represent lived experience.[3]

Merleau-Ponty eulogizes Cézanne's painting for revealing an extremely careful interpretation of nature that "is not a reflection distinct from the act of seeing."[4] Paintings that follow a geometric or photographic perspective engage with nature in a way that prioritizes "understanding," that takes sensory data as the base on which ideas of scientific and mathematical exactness can then be constructed.[5] In Raphael's School of Athens, perspective commands the composition's physical subject – the understanding supplants the senses, the form supplants the matter. As a phenomenologist Merleau-Ponty argues that this prioritization of the rational is misplaced: our body is constitutive of our ontology, so it is crucial that we trust our senses. Cézanne's painting honors this position: it depicts "matter as it takes on form, the birth of order through spontaneous organization."[6] Cézanne did not show "nature pure" to paint like a "savage," unskilled artist; rather, he wanted to put "intelligence, ideas, sciences, perspective, and tradition back in touch with the world of nature [...] from which they came."[7]

Cézanne's painting represents a movement "back to nature," says Merleau-Ponty, because unlike paintings that follow geometric perspective, his use of form conveys the pure sensuousness of nature. He contends that without using outline or perspectival arrangement, Cézanne managed to relate the "imperious unity," the "presence," and the fullness of the world as

1   Merleau-Ponty, Maurice. "Cézanne's Doubt." In The Merleau-Ponty Aesthetics Reader. (Evanston: Northwestern University Press, 1993), 63.
2   Ibid., 64.
3   Ibid., 67.
4   Ibid., 66.
5   Ibid., 64.
6   Ibid., 63.
7   Ibid., 64.

we experience it.[8] Cézanne did not distinguish color from outline when he painted; instead, in his painting the spatial structure of the picture "vibrat[es]" as it emerges in a "movement" combining both outline and color.[9] Tracing a single outline would otherwise "sacrific[e] depth," claims Merleau-Ponty, as it does not capture the fact that an object is "an inexhaustible reality full of reserves."[10] To properly convey the myriad facets of a given object, the painter must therefore depict "the swelling of the object" through modulated colors that indicate several outlines, thereby expressing the object's solidity and materiality.[11] Cézanne's painting achieves this goal.[12] The object is no longer "hidden," no longer lost in "reflections;" it presents itself in its "true density," as if it was "subtly illuminated from within," beckoning the viewer to discern it.[13]

Cézanne's painting, Merleau-Ponty argues, resembles the world as we perceive it by embodying the process of "becoming" involved in every relation (every act of intentionality) between us and the world. Most of us, "forgetting the viscous, equivocal appearances" of objects in the world, "go through them straight to the things they present."[14] Because our natural attitude leads us to believe in the "imposed order" of objects, we see them as being static and defined, we "agree about them" and are "anchored in them."[15] This attitude makes the world appear familiar, reassuring, and easily navigable. However, Merleau-Ponty insists, this does not correspond to our lived experience. Perception is not static or ready at hand; rather, it is movement, "chaos," the strange process of "object[s] in the act of appearing," of the world "becoming" the world.[16] Since perception is unsettling and discomforting, the painter who seeks to capture its true character can only feel one emotion – the "feeling of strangeness – and only one lyricism – that of the continual rebirth of existence."[17] Merleau-Ponty believes Cézanne's painting encapsulates this

8   Ibid., 65.
9   "As you paint, you outline; the more colors harmonize, the more the outline becomes precise," Cézanne once remarked. (Ibid).
10   Ibid.
11   Ibid., 65.
12   To illustrate this point, Merleau-Ponty compares Cézanne's color palette to that of the Impressionists: while the latter consists of seven colors, Cézanne's is composed of eighteen colors, including warm tones and shades of black (Ibid 62). This, Merleau-Ponty suggests, is evidence that Cézanne wanted "to make of Impressionism 'something solid'"(Ibid 63). Whereas in Impressionism the object fades behind the light and atmospheric effects dominate the composition, Cézanne's paintings "find it again behind the atmosphere" (Ibid 62).
13   Ibid.
14   Ibid., 67.
15   Ibid., 64.
16   Ibid., 65.
17   Ibid., 68.

Cézanne did not distinguish color from outline when he painted; instead, in
his painting the spatial structure of the picture "vibrates" as it emerges in a
"movement" combining both outline and color.

Raphael's masterpiece exemplifies geometric perspective: all the figures and
architectural structures recede toward and are scaled according to an invis-
ible vanishing point, located at the exact center of the composition.

truth, which is why the people in his pictures are strange, almost inhumane, "as if viewed by another species."[18]

Despite these elements Merleau-Ponty suggests that Cézanne – and all other painters for that matter – will never be able to fully capture in painting our "lived perspective."[19] This is because the painter's project is to convey the "imperious unity, the presence, the unsurpassable plenitude" of our experience of the world – she must translate onto the canvas every aspect of our sensory experience.[20] Yet this project rests on a paradox: one cannot represent all the sensations, all the relations and all the possibilities of the world by employing strictly visual means.[21] Cézanne knew this when he granted that he "freeze[s]" the spontaneous movement of perception the moment he re-paints it on the canvas.[22] Merleau-Ponty argues that Cézanne was extremely frustrated by this powerlessness in the face of wanting to portray the world completely – of wanting to recreate a piece of nature, and not merely imitate it – but not possessing the God-like omnipotence required for such a project.[23] Merleau-Ponty even posits that Cézanne's solitude and instability could not be explained strictly by his schizoid temperament but instead by the purpose of his work, which called him to undertake an impossible metaphysical task.[24]

Merleau-Ponty outlines another paradox at the heart of the painter's quest to capture our "lived perspective" on the canvas: the "phenomenon of expression."[25] The process of artistic expression is paradoxical for artists like Cézanne, Merleau-Ponty points out, because they seek to crystallize the lived experience(s) of a multitude of people by relying solely on their own personal experience. When he shares his work for others to see, the artist has no way of knowing whether his work will resonate with other people or whether it will fall on deaf ears. Merleau-Ponty even compares the artist "launch[ing]" his work into the social sphere to the man who "once launched the first word, not knowing whether it will be anything more than a shout [or] whether it can detach itself from the flow of individual life in which it was born."[26] If she is successful, the artist's work "will have united separate lives" and given "an identifiable meaning"

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18   Ibid., 66.
19   Ibid., 64.
20   Ibid., 65.
21   Ibid.
22   Ibid., 64.
23   Ibid., 70.
24   Ibid., 69.
25   Ibid., 71.
26   Ibid., 69.

to something that multiple individuals, and even potentially future generations, will be able to share.[27] The great work of art, Merleau-Ponty insists, "will no longer exist in only one [consciousness] like a stubborn dream or a persistent delirium, nor will it exist only in space as a colored piece of canvas. It will dwell undivided in several minds, with a claim on every possible mind like a perennial acquisition."[28] Nevertheless, even the greatest artwork cannot bridge the gulf between the universal and the individual. Even if it succeeds in "resonating" with every person in the world, the artwork will never be able to create a universal meaning that successfully aggregates the infinite particular meanings it has inside of every individual consciousness. Art can convey a shared meaning – a meaning that allows an overlap between several consciousnesses – but this shared meaning will never perfectly map onto each individual's lived experience. Art can only ever be the fractional center of a Venn diagram comprised of billions of circles.

Thus, painting can only ever partially resemble the world that we see with our eyes according to Merleau-Ponty because expressing what exists – and the countless meanings that come with it – is an "endless" task that "must satisfy an infinite number of conditions."[29] Even though Cézanne's painting conveys the sensuousness of the nature he was trying to recreate, as well as the "impression of an emerging order […] organizing itself before our eyes" that happens in perception, his art – or any other artist's – will never be able to actually become a piece of nature, will never be able to embody lived experience.[30] There will always be a tension between the personal, individual nature of artistic expression and its goal of arriving at a universal meaning. Nonetheless, Merleau-Ponty concludes, these limitations have never stopped us from creating and engaging with art. Even though our comprehension of any artwork is bound to be merely partial, our humanity will always compel us to try to see "the landscape in its totality and in its absolute fullness."[31]

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27   Ibid., 70.
28   Ibid.
29   Ibid., 65-66.
30   Ibid., 65.
31   Ibid., 67.

Merleau-Ponty, Maurice. *"Cézanne's Doubt".*
In The Merleau-Ponty Aesthetics Reader,
edited by Galen Johnson, 59-75. Evanston:
Northwestern University Press, 1993.

Frege's Abandonment of Logicism

Austen Friesacher

Much of Frege's work on mathematics was concerned with deriving arithmetic from solely logical means. In 1893 Frege published his *Grundgesetze*, in which he provided axioms which could be used to derive what we now call the Dedekind-Peano axioms of arithmetic. In doing so, Frege thought he had vindicated logicism once and for all. Bertrand Russel soon dashed Frege's hopes by showing how one could derive a paradox from his axioms, relying especially on his "Basic Law V". Strictly speaking, Russel's paradox only implies that Frege's particular axiomatization is inconsistent, yet Frege responded by writing, "The collapse of my Law V seems to undermine not only the foundations of my arithmetic but the only possible foundations of arithmetic as such". By saying that the logicist project as whole is untenable, Frege goes beyond the immediate result of Russell's paradox. This essay seeks to explain why Frege drew this stronger conclusion. In doing so, we will be answering three questions: why Frege thought Hume's principle required proof, why Frege thought Hume's principle could not be proved from logic, and why Frege was committed to abstraction principles.

1 Background

Upon the discovery of Russell's paradox, Frege wrote, "The collapse of my Law V seems to undermine not only the foundations of my arithmetic but the only possible foundations of arithmetic as such"[1]. This conclusion goes beyond the immediate result of Russell's paradox: Frege is not only acknowledging that his particular axiomatization of arithmetic is inconsistent, but that the logicist project as whole is untenable. This essay seeks to explain why Frege drew this broader conclusion. In doing so, we will be answering three questions: why Frege thought Hume's principle required proof, why Frege thought Hume's principle could not be proved from logical means alone, and why Frege was committed to abstraction principles. To motivate the discussion we give a brief overview of Frege's project and the tech-

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1   Frege, Gottlob, Hans Hermes, and Peter Long. 1997. Posthumous Writings. Oxford: Blackwell. p.132

nical results which have recently arisen. In particular, we will emphasize that Hume's principle (HP), when combined with standard second-order logic, is sucient to derive Frege's laws of arithmetic. We will then remark with Heck that Frege appears to have considered adopting HP as an axiom, which raises our rst question. Then we will draw on Hallett and Heck to show why concerns about referential xity and the Caesar problem precluded Frege from taking Hume's Principle as an axiom. Next we will exposit Blanchette's argument in "The Breadth of the Paradox" to explain why Frege thought Hume's principle was unprovable. Finally, we will show why Frege's restriction on existential proofs necessitated the use of abstraction principles and we will then remark that the arguments presented above regarding Hume's principle can be generalized to preclude all abstraction principles.

In the *Grundlagen*, Frege states his goal of deriving the laws of arithmetic from solely logical means and then proceeds to outline his approach. The proof sketches presented in the *Grundlagen* were later formalized in his *Grundgesetze*, which sought to prove what the *Grundlagen* "made probable"[2]. There, Frege derives statements equivalent to the Dedekind-Peano axioms from the axioms of what is called "Frege Arithmetic" (FA). The axioms of FA are equivalent to the standard axioms of second-order logic plus his infamous "Basic Law V" (BLV), which states:

$$(\grave{\epsilon}\phi(x) = \grave{\epsilon}\psi(x)) = (\forall x(\phi(x) = \psi(x)))$$

As expressed above, BLV states the equivalence[3] between an identity of value-ranges[4] and two functions' agreement on all values of a variable. In other words, BLV states that if two functions have the same truth-value for all arguments, then their value-ranges are the same, and vice versa. But introducing value-ranges gets Frege into trouble as it leads him to contradiction. Russell's paradox shows that, by creating the set of all sets which don't contain themselves, one arrives at a paradox where the set contains itself if and only if it does not contain itself. An analogous construction can be made for functions and value-ranges which, for Frege, means that value-ranges he uses

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2   Frege, Gottlob, and J.L Austin. 1980. The Foundations Of Arithmetic. 2nd ed. Evanston Ill: Northwestern Univ. Press. p.62

3   By the time of the *Grundgesetze*, Frege thought sentences' referents were truth-values, which is why the above equivalence statement is expressed as an identity.

4   In the *Grundlagen* Frege talks of concept's extensions but in the *Grundgesetze* switches to talking about function's value-ranges. Here we'll use the value-range terminology.

in BLV aren't guaranteed to exist, on pain of contradiction. But if the value-range's referenced in BLV don't exist, then its terms fail to refer and the statement as a whole is meaningless. Thus, Frege had to abandon his theory of value ranges. Recent results have shown that this is less damaging than was originally thought.

Central to Frege's project is the statement known as "Hume's Principle". Hume's Principle is an abstraction principle which states that the number of F's is equal to the number of G's i F is equinumerous with G and can be formalized as:

$$(\#F = \#G) = (F \approx G)^5$$

For reasons which will be explained below, Frege derived HP from BLV and an explicit denition of numbers as value-ranges. But, while Frege implicated value-ranges in order to derive HP, Boolos observed that after sketching a proof for HP, Frege makes no further use of his theory of value-ranges in the *Grund-lagen*.[6]Heck has since extended this result to the *Grundegesetze* and shown that the only essential use of value-ranges in *Grundegesetze* is his derivation of HP.[7] This means that, after proving HP, Frege did not need to use BLV or his explicit denition of number. Since value-ranges were what lead to Russell's paradox, this suggests that the axioms of second-order logic plus HP may be consistent. Indeed, it's been shown that FA is consistent relative to second-order arithmetic and so its inconsistency would be very surprising, to say the least. While Frege could not have known all these results, Heck argues that there is reason to suspect that he knew HP was formally sucient to derive the desired results.[8] After acknowledging the problems associated with BLV, Frege suggests using something rather reminiscent of HP:

> We can also try the following expedient, and I hinted at this in my Foundations of Arithmetic. If we have a relation $\phi(\xi,\zeta)$ for which the following propositions hold: (1) from (a; b) we can infer (b; a), and (2) from $\phi(a; b)$ and $\phi(b; c)$ we can infer $\phi(a; c)$; then this relation can be transformed into an equality (identity), and $\phi(a; b)$ can be replaced by

5   Frege, Gottlob, and J.L Austin. 1980. The Foundations Of Arithmetic. 2nd ed. Evanston Ill: Northwestern Univ. Press. pp.63,73

6   Boolos, George, John P Burgess, and Richard Jerey. 1999. Logic, Logic, And Logic. Cambridge (Massachusetts): Harvard University Press. ch.11

7   Heck, Richard G. 2011. Frege's Theorem. Oxford: Oxford University Press. ch.2

8   Ibid. p.11

writing, e.g., §a = §b. If the relation is, e.g., that of geometrical similarity, then "*a* is similar to *b*" can be replaced by saying "the shape of *a* is the same as the shape of *b*". This is perhaps what you call "denition by abstraction".[9]

But if Frege knew that HP is sucient, why did he take Russell's paradox to mean the end of logicism?

2 Why HP Can't be an Axiom

The rst step in understanding why Frege abandoned logicism is understanding why he insisted on a derivation of HP. Modern neo-logicists, inspired by the formal suciency of HP, often advocate adopting it as an axiom. They then argue that a derivation of the axioms of DPA from FA would vindicate logicism. Heck suggests that Frege considered this approach himself, but ultimately found it impassible due to the Caesar Problem (CP). Similarly, Hallett shows why concerns with referential xity precluded Frege from taking HP as an axiom.

2.1 Hume's Principle and Fixed Reference

In his correspondence with Hilbert, Frege expressed a number of methodological disagreements. Many of these boiled down to an insistence on xed reference. Michael Hallett has shown how this insistence led Frege to reject HP as an axiom, and hence to introduce BLV and his explicit denition of number. One of Frege's main issues with Hilbert's formalism is that sentences and terms do not have xed reference. Multiple interpretations of terms may be consistent and so instead of having one meaning, a given sentence could have many. But Frege thought that if something did not have one particular meaning, then it did not have any meaning. So if a sentence lacked referential xity then it was meaningless and hence not true.[10] This is problematic for HP as an axiom because Frege took axioms to be sentences which expressed true Thoughts; so if HP lacked referential xity then it could not be true and hence could not be an axiom. But, if any terms in a sentence lack referential xity, then so does the sentences as a whole: xed reference ows upwards from terms to sentences.[11] So if Frege wanted to take HP as an axiom, he'd

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9   Frege, Gottlob, Hans Hermes, and Peter Long. 1997. Posthumous Writings. Oxford: Blackwell. p.141

10   Hallett, Michael. 2010. "Frege And Hilbert". In The Cambridge Companion To Frege, 413-464. Cambridge University Press. p.431

11   Frege acknowledged that not all terms can be dened and ultimately all systems require primitive terms. But, Frege maintained that primitives could have their reference xed by use of "elucidations". This approach is problematic but

have to x its reference by xing the reference of its terms, "propositions (axioms, fundamental laws, theorems) must not contain a word or sign whose sense and meaning, or whose contribution to the expression of a thought, was not already completely laid down".[12] But, HP makes mention of 'number', so unless the reference of 'number' is xed, HP fails as an axiom.

In order to explain how numbers' references are xed, Frege briefly considers the possibility that numbers are primitive and that their meanings are given directly, but he promptly rejects this.[13] Given this, the next step for Frege would then be to dene number in terms of other terms which have their reference xed (either because they are primitives or because they can dened by terms with xed reference). Once the terms have been dened explicitly, one can infer a "self-evident" proposition which expresses the equivalence of deendium and deniens:

> Every denition contains a sign (an expression, a word) which had no meaning before and which is rst given a meaning by the denition. Once this has been done, the denition can be turned into a self-evident proposition which can be used like an axiom.[14]

But Frege needs to do more than just x numbers' references, he needs to ensure that they're "logical objects", or objects who we know solely through logic. For Frege, the only way we can understand numbers as logical objects is to understand them in terms of value-ranges, "how do we apprehend logical objects? And I have found no other answer to it than this; we apprehend them as extensions of concepts, or more generally, as ranges of values of functions".[15] This is what gave rise to his explicit denition of numbers as value-ranges. But, as we've seen, his theory of value-ranges is inconsistent and untenable. So after the discovery of Russell's paradox, Frege was unable to give an explicit denition of numbers which xes their reference and explains their status as logical objects.

A seemly plausible approach then is to use HP as implicit denition of numbers. Indeed, this is the purpose for which Frege

that's beyond the scope of this essay. For more see Hallett 2010

12   Frege, Gottlob, and Gottfried Gabriel. 1980. Philosophical And Mathematical Correspondence. Chicago: University of Chicago Press. p.36

13   Frege, Gottlob, and J.L Austin. 1980. The Foundations Of Arithmetic. 2nd ed. Evanston Ill: Northwestern Univ. Press. p.62

14   Frege, Gottlob, and Gottfried Gabriel. 1980. Philosophical And Mathematical Correspondence. Chicago: University of Chicago Press. p.36

15   Ibid. p.141

originally suggests HP. However, Frege ultimately rejects this approach. Hallett explains that using HP as an implicit denition once again raises problems of referential xity for Frege. Hume's principle doesn't suciently restrict the possible meaning of numbers and in doing so leaves interpretations which would undermine his logicist project. Even if the terms of HP were xed so that it had a determinate extension, within that extension it would not be certain that everything is a number. Hallett observes that if one reformulates HP as Boolos does:

$$\exists f \forall F, G [f(F) = f(G) \leftrightarrow F \approx G]$$

then it becomes clear that an existential statement is being made, with little stipulation about what sort of things instantiate it.[16] Thus, "if HP were taken as a primitive truth (axiom) and yet yields no direct knowledge of the kind of things numbers are, then we have no guarantee that the numbers are 'logical' objects".[17] But if numbers aren't understood as logical objects then arithmetic is not a result of logic and logicism fails. Another more famous reason that HP fails as an implicit denition is the Caesar problem. For this we will turn to Heck.

2.2 Hume's Principle and the Julius Caesar Problem

In "Julius Caesar and Basic Law V", Heck sets out to explain why Frege didn't take HP as an axiom or implicit denition of number. He motivates the discussion by drawing on textual evidence to argue that Frege was aware that HP is sucient to derive his laws of arithmetic (so there's no formal reasons preventing him from adopting HP) and that he even considered taking it as an axiom. Heck then argues that, "What ultimately forces Frege to abandon his logicism is his inability to resolve the Caesar problem".[18]

According to Heck, what Frege nds so troubling about the Caesar objection is that it shows that HP fails to explain how we understand numbers. As declared in his famous "context principle", Frege seeks to explain our understanding of numbers by explaining our understanding of sentences about them.[19] This is precisely what HP is intended to do. However, HP only explains some of our uses of numbers. HP does not explain mixed-identity statements,

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16   Hallett, Michael. 2010. "Frege And Hilbert". In The Cambridge Companion To Frege, 413-464. Cambridge University Press. p.442
17   Ibid. p.423
18   Heck, Richard G. 2011. Frege's Theorem. Oxford: Oxford University Press. p.115
19   Frege, Gottlob, and J.L Austin. 1980. The Foundations Of Arithmetic. 2nd ed. Evanston Ill: Northwestern Univ. Press. intro.

sentences of the form: "*n is* equal to the number of *F*s". Thus HP does not tell us whether or not Caesar is a number, nor whether 5 is a number. It only tells us when there are the same number of *things*. Frege remarks that, of course, no one actually confuses Julius Caesar for a number, but he explains that that's not because of HP.[20] If there's something beyond HP that helps us realize that Caesar isn't a number, then either we're relying on intuition, or there's another denition that precludes Caesar from being a number: "there must be *more* to our apprehension of numbers than a mere recognition that they are objects that satisfy HP".[21] Since Frege could not admit intuition into his system, he needed to supplement HP to explain how we 'apprehend' numbers as logical objects and how we know Caesar isn't a number.

Frege's appeal to Basic Law V and an explicit denition of numbers as value-ranges was precisely the supplement intended to resolve the Caesar objection. By dening numbers in terms of value-ranges Frege sought to explain how we understand sentences using numbers, while preserving their status as logical objects: "how do we apprehend logical objects? I have found no other answer to it than this, We apprehend them as extension of concepts".[22] It may seem that this does not so much solve the Caesar problem as it does defer the problem. It's not clear why BLV is immune from a Caesar problem of its own: how do we know Julius Caesar isn't a value-range? Frege's solution to this is simply that knowledge of value-ranges is fundamental: Frege is "assum[ing] it is known what the extension of a concept is"[23], saying that, "we must regard it as a fundamental law of logic that we are justied in thus recognizing something common to both and that accordingly we may transform an equality holding generally into an equation (identity)".[24] Thus by deriving HP from a foundation of value-ranges, Frege sought to protect HP from the Caesar problem. Since we know that Caesar isn't a value-range, and since numbers are value-ranges, Caesar can't be a number. More generally, we know whether or not *n* is a number because we know whether or not *n* is a value range of a certain type, and because we think of numbers as a certain type of value-range.[25] However,

20   Frege, Gottlob, and J.L Austin. 1980. The Foundations Of Arithmetic. 2nd ed. Evanston Ill: Northwestern Univ. Press. p.66
21   Heck, Richard G. 2011. Frege's Theorem. Oxford: Oxford University Press. p.121
22   Frege, Gottlob, Hans Hermes, and Peter Long. 1997. Posthumous Writings. Oxford: Blackwell. p.140-141
23   Frege, Gottlob, and J.L Austin. 1980. The Foundations Of Arithmetic. 2nd ed. Evanston Ill: Northwestern Univ. Press. p.68 note
24   Frege, Gottlob, Philip A Ebert, and Marcus Rossberg. 2014. Basic Laws Of Arithmetic. Oxford: Oxford University Press. v.2 p146
25   In the *Grundlagen* Frege denes "the number which belongs to the concept F

Frege's resolution of the Caesar problem meant that HP is not an axiom but is derived from his theory of value ranges. This means Frege needs to nd a way to derive HP which avoids the problems of Russell's paradox.

3 Why HP Can't be Proved

So far we've seen why Frege rejected HP as an axiom, but of course if one could derive HP from logic, logicism would be vindicated | this was Frege's approach. But while Frege's particular attempt to derive HP failed, it was not shown that all attempts to prove HP would fail. Russell's paradox serves as a local counter-example, not a global one, so it does not eliminate the possibility of proving HP. Despite this, Frege declared that he thought his logicist project was impossible. Blanchette explains that this declaration was due to Frege's restrictions on existential proofs.

Though Russell's paradox shows that Basic Law V is inconsistent with the axioms of second-order logic, it does not show that all abstraction principles are similarly inconsis- tent. In fact, as was remarked earlier, HP is consistent relative to second-order logic. But nonetheless, Russell's paradox undermines all abstraction principles. While some abstraction principles may be compatible with the standard axioms, it's not clear how to distinguish the 'good' principles from the 'bad' ones *a priori.* Providing *a priori* support of the 'good' principles is exactly Boolos's "bad company objection".[26] For Frege to prove HP he would have to answer Boolos's bad company objection for the case of HP.[27] Blanchette shows that Frege's restrictions on existential proofs leave him unable to respond to it for any abstraction principle. She does so by demonstrating that contemporary approaches tothe problem are not available to Frege given his philosophy. A modern approach to the bad company objection is to propose a principle along the lines of (CON): "If an abstraction principle is consistent then it's true". But Blanchette observes that, for Frege, there are two issues with this approach. First, Frege explicitly rejects inferences from consistency to truth or instanti-

is the extension of the concept "equal to F"". This can then be implemented in mixed-identity statements as in the example, "0 is the number which belongs to the concept "not identical with itself"". Frege inductively denes all natural numbers, making this feasible for any n (*Grundlagen* pp.68,77)

26   Boolos, George, John P Burgess, and Richard Jerey. 1999. Logic, Logic, And Logic. Cambridge (Massachusetts): Harvard University Press. p.311-2

27   More precisely, Frege would have to prove HP without the use of other abstraction principles, or have proved those abstraction principles as well. Boolos shows how HP follows from a principle he calls "Numbers", but Numbers is expressed as an existential statement and so isn't clearly logical.

ation.[28] This is made evident in his correspondence with Hilbert when he says:

> Suppose we knew that the propositions
> (1) A is an intelligent being,
> (2) A is omnipresent,
> (3) A is omnipotent,
> together with all their consequences did not contradict one another; could we infer from this that there was an omnipotent, omnipresent, intelligent being? This is not evident to me. . . I cannot accept such a method of inference from lack of contradiction to truth.[29]

Further, any proof that an abstraction principle is consistent would rely on model-theoretic methods. But, Frege also rejects model theory, so he couldn't implement (CON) even if he accepted it.[30] These two issues extend to other potential solutions to the bad company objection.

Blanchette considers the rule-scheme (S): "If an abstraction principle meets the safety condition, then it is true and its introduced singular terms refer"[31], where the safety condition is specied and supposed to guarantee 'goodness'. While this approach is sometime used now, Frege would reject it for reasons similar to those which lead him to reject (CON). Blanchette claims that for Frege to accept (S), there would need to be an argument for why satisfaction of the safety condition implies existence and consistency, but Blanchette asserts that this is not possible. Once again, even if Frege accepted (S), he wouldn't have been able to implement it without model theory.[32]

4 The Necessity of Abstraction Principles and the Necessity of Their Failure

Considering Frege's restrictions on existential proofs, one may

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28   Blanchette, Patricia. 2015. "The Breadth Of The Paradox". Philosophia Mathematica 24 (1): 30-49. doi:10.1093/philmat/nkv038. p.38
29   Frege, Gottlob, and Gottfried Gabriel. 1980. Philosophical And Mathematical Correspondence.Chicago: University of Chicago Press. p.46
30   Blanchette, Patricia. 2015. "The Breadth Of The Paradox". Philosophia Mathematica 24 (1): 30-49. doi:10.1093/philmat/nkv038. p.36
31   Ibid. p.39
32   Or at least Blanchette asserts this. While it seems plausible, it's not immediately clear that model-theory would be necessary. Nothing is said about safety conditions so it's not clear how Blanchette infers the fact that satisfying them would require model-theory. But, either way, Frege would still need a safety-principle whose satisfaction ensure existence.

wonder how he ever justied his proofs of existence. The answer is abstraction principles. Frege used abstraction principles to introduce term-forming operators and held that some abstraction principles were self-evidently justied. Frege said because each side of the biconditional has the same sense, the abstraction principles are simply "re-carving" the content: "the equivalence bertween the two sides of (relevant) abstraction principles is self-evident, something that can be doubted only by those who are confused about the semantics of the singular terms in question".[33] Frege bemoaned mathematicians who "list properties and then say: we create a thing that has these properties"[34], but thought that our ability to identify something common to multiple things and name that common thing was primitive, "the generality of an equality is here converted into an equality (identity). . .We do it in full awareness and by appealing to a basic law of logic".[35] However, this justication of abstraction principles is rejected along with BLV by Frege after Russell's paradox | thus calling into doubt Frege's only method of existential proofs. If Frege is to save logicism, he must recover the ability to prove existence. To do so he would need to justify the use of at least some abstraction principles.

As was mentioned earlier, justifying the use of particular 'good' abstraction principles is precisely Boolos' bad company objection. Blanchette argued that Frege cannot resolve the bad company objection in the case of HP and BLV, but her arguments can be generalizedto abstraction principles in general. If Frege wanted to defend abstraction principles then he would need a safety-principle of sorts, and he would need an argument that shows satisfaction of his safety principle ensures existence. Further, Frege would need a safety-condition whose satisfaction could be demonstrated without model theory. Neither of these two arguments are particular to BLV or HP and so hold for all abstraction principles, thus calling into doubt Frege's only hope. But this argument is broader than it needs to be. Blanchette's arguments show why Frege cannot utilize any abstraction principles, but it can also be seen that Frege needed an abstraction principle specically about value- ranges. Frege rejected HP's status as an axiom because he could not resolve the Caesar problem: because he could not explain how we understood numbers as primitive logical objects. The only way Frege saw of explaining our understanding of number, and

33   33Blanchette, Patricia. 2015. "The Breadth Of The Paradox". Philosophia Mathematica 24 (1): 30-49. doi:10.1093/philmat/nkv038. p.44
34   Frege, Gottlob, Philip A Ebert, and Marcus Rossberg. 2014. Basic Laws Of Arithmetic. Oxford: Oxford University Press. v.1 14p
35   Ibid. p.147

the only way he thought we understood logical objects, was through appeal to value-ranges. But if value-ranges are necessary, and they must be introduced through abstraction principles (as that was Frege's only method of existential proof), then there must be an abstraction principle governing value-ranges. This is precisely what the multiple iterations of BLV tried to do, but all failed. If Frege wanted to rescue logicism he would have needed an abstraction principle introducing value-ranges which was consistent with the other axioms of second-order logic. After multiple promising but failed attempts, it is understandable why Frege thought the project hopeless.

5 Conclusion

In the wake of Russell's paradox, Frege thought his goal of deriving the laws of arithmetic from logic alone was impossible. This is because he (1) refused to adopt Hume's principle as an axiom, (2) recognized that he could not prove Hume's principle and meet Boolos's bad company objection, and because (3) the only hope he had of satisfying his methodological constraints and resolving the Caesar problem were ruled out by Russell's paradox. Hume's principle could not be taken as an axiom because it did not have a xed reference and hence needed derivation in order achieve xed reference. Similarly, Hume's principle was insucient as an implicit denition of numbers as it did not fully explain our use of number words and so did not explain how we understand numbers as logical objects. The only way to resolve the Caesar problem was to dene numbers in terms of value-ranges, but this is untenable. Since BLV undermined the validity of abstraction principles, HP needed to be proved from axioms which are not abstraction principles. But this is not possible as doing so would require an argument for existence *eh nihilo*, and, even if this was provided, Frege would (most likely) need model-theory to apply it. Finally, we saw that, prior to the discovery of the paradox, the only method of existential proof which Frege endorsed was through self-evident abstraction principles and that was necessarily abandoned after the paradox. Additionally, Frege would have required abstraction principles governing value-ranges, but his theory of value-ranges had proved to be persistently problematic.

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Leibniz's Account of Causation: The Communication of Active Individual Substances

Thea Koper

The aim of this paper is to demonstrate Leibniz's objection to the occassionalist view (which takes substances to be entirely passive entities, moveable only by God) put forward by Malebranche and to articulate the reasons for his objection. The paper illustrates Leibniz's own metaphysical picture and examines his effort to discover what gives the appearance of a well-ordered, harmonious world in which mind and body seem to communicate. Leibniz's objection to the occasionalist view arises out of a certain predicament. It is Leibniz's realization that, up until this point, he has failed to find a satisfactory proof for the union of body and soul, or the apparent communication between substances. It seems as if this predicament emerges out of Leibniz's own metaphysical picture, that is, his view that universes are comprised only of individual substances that have been created by God. These substances contain in themselves all reality and seem to have no need to communicate with each other, since all phenomena naturally flow from and are contained in the complete notion of each individual substance. Nevertheless, Leibniz remains unsatisfied with his own metaphysical picture at this point, since he admits that, so far, it fails to account for the apparent interaction between substances.

Leibniz's wish is then to discover what accounts for, or what gives us the appearance of, a well-ordered world in which mind and body are in sync. After explaining how the views of Descartes and his followers, as well as the occasionalists, fail to eliminate the difficulty which is implicit in the communication of substances, Leibniz reveals his own views on the matter. What makes Leibniz's account distinct from the occasionalist picture has to do especially with his view on the role of philosophy and his conception of substance.

The aim of this paper is twofold: firstly, it will explore Leibniz's reasoning for his objection to "the system of occasional causes",[1] as put forward by Malebranche in The Search After Truth; and secondly, it will examine Leibniz's own alternative account of causation, which is derived mostly from his understanding

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1   G. W. Leibniz, Philosophical Essays, ed. trans. Roger Ariew and Daniel Garber (Indianapolis: Hackett Publishing Company, 1989), 143.

of what philosophy's goal should be as well as his conception of substance.

The occasionalist view, as recapitulated by Leibniz in "A New System of Nature", claims that substances are ultimately passive and unable to act on their own accord. What gives the impression then of the communication between body and soul, as well as the appearance of a well-ordered world, according to the occasionalists, is God. God is the underlying, fundamental cause of all things.[2] Malebranche states that there can only ever be "one true cause because there is only one true God".[3] It follows that substances are moveable only through God since He is the true cause of everything, and hence the only one who can "cause thoughts to arise in the soul on the occasion of motions of matter".[4] According to Malebranche, there is nothing in substances themselves to account for their movement or action in the world, since motion is "nothing other than the will of God".[5] A rolling ball that collides with another ball is thus only a 'natural' or 'occasional' cause, acting through the force of the will of God.[6] To be clear, Malebranche does not deny the effects of these natural causes in the material world, however, he is firm and unwavering in his claim that these natural causes do not contain in themselves any real power or force to produce effects. It is precisely with this last statement that Leibniz takes issue.

Leibniz's objection to Malebranche's view is rooted in the allegation that Malebranche unduly appeals to what is called a "Deus ex machina".[7] Leibniz thinks that the simple reliance on God as the general cause of all things is an insufficient means of explanation and justification when it comes to dealing with problems in the natural world. It is important to note, however, that Leibniz does not in fact find anything fundamentally wrong with the occasionalist view that substances do not directly influence one other. Indeed, it is entirely consistent with Leibniz's metaphysical picture to conceive of substances as individual, independent entities. To be clear, Leibniz states that only when speaking in purely "metaphysical rigor"[8] does it suffice to say that "there is no real influence of one created substance on another, and that all things, with all their reality, are continu-

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2   Ibid.
3   Nicolas Malebranche, The Search After Truth, trans. Thomas M. Lennon
and Paul J. Olscamp (Columbus: Ohio State University Press, 1980), 448.
4   Leibniz, Philosophical Essays, 143.
5   Malebranche, The Search After Truth, 448.
6   Ibid.
7   Leibniz, Philosophical Essays, 143.
8   Ibid.

ally produced by the power [vertu] of God".[9] In a metaphysically abstracted sense, Leibniz does hold a view similar to the occasionalists, however, this is only part of the picture. Leibniz ultimately objects to Malebranche's occasionalism since it makes excessive use of the general cause (ie. God), which turns out to be insufficient when it comes to addressing real problems of science and physics. While Leibniz undoubtedly takes God to be the fundamental cause of all things, he believes that it is trivial and insufficient to appeal only to the general, true cause. Leibniz likens such an action to "having recourse to [a] miracle".[10]

Leibniz makes his objection to the occasionalist view evident through his conception of what the role of philosophy is. Leibniz states that "in philosophy we must try to give reasons by showing how things are brought about by divine wisdom, but in conformity with the notion of the subject in question".[11] If everything was simply passive matter being acted upon by God, whether or not through the occasion of motion, then there would not be anything in nature to study. There would be no point to science or physics, or even philosophy if this were the case. This is where the notion of "secondary causes" comes in, or "natural" causes, as Malebranche calls them. Leibniz believes that it is entirely plausible to think that God should "give a substance, from the beginning, a nature or an internal force that can produce in it [...] everything that will happen to it [...] without the help of any creature".[12] In other words, the idea of substances containing in their own nature all of their phenomenal reality in no way undermines God's divine wisdom. In this way, it is entirely possible to both acknowledge that God is the general cause of the universe and consequently of all substances, and also to study the nature of those substances in question, ie. the secondary causes. Thus, while Leibniz agrees with the occasionalists that substances are independent and have only God as their creator, he distinguishes his view by recognizing that it is still possible to delve further into the nature of substances as secondary causes in order to ascertain whether or not there is communication between them. In order to explore Leibniz's views on the matter, we must first explain what he means by a 'substance.'

Leibniz's understanding of what constitutes a substance arises from his dissatisfaction with the mechanistic view of the world.

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9   Ibid.
10   Ibid.
11   Ibid.
12   Ibid., 144.

Leibniz's engagement with mathematics and physics prompted his realization that "it is impossible to find the principles of a true unity in matter alone",[13] since matter is infinitely divisible. From this statement one of Leibniz's main assumptions becomes clear: that unity is the necessary element for the configurations of phenomenal reality. Nevertheless, as Leibniz puts it, "a multitude can derive its reality only from true unities, which have some other origin".[14] Thus, Leibniz found himself forced to leave behind the world of matter in search for the origin of true entities in the realm of metaphysics. Such entities he discovered do exist—as pure unities, as substances.

In order to account for the appearance of order in our universe, as well as how it seems that substances interact with each other, Leibniz advances his complete notion principle. Put briefly, this principle claims that every substance has a complete notion which contains all of its predicates. This simply means that any substance (e.g. a human being) already contains within itself everything that it is and has been and will become and/or effectuate in the future. Everything that exists for that substance, namely the entire universe, is contained within its own notion (hence the term complete notion, where notion refers simply to the substance). In other words, every substance, which is created by God, contains all of its phenomenal reality from the time of its creation. Substances thus have this "perfect spontaneity" which is both relative to the substance's original constitution and in "conformity relative to external things".[15] Thus, all it takes is the soul, which Leibniz understands as a substance, to spontaneously will the body to act in order for the body to act on its own and in accordance with the laws of nature.[16] In this way Leibniz claims that there is communication between the substances, and hence, "the union of soul and body".[17] Since all substances contain within their notion all reality, it can be said that each substance represents the whole universe in its own way. Since every substance behaves as a world apart, spontaneous and self-sufficient, it must therefore take a "perfect agreement among all these substances" for there to be order and harmony in the world as we experience it. This idea of agreement, of preestablished harmony, is thus what gives rise to the appearance of communication between substances, and the consequent union of body and soul.

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13 Ibid., 139.
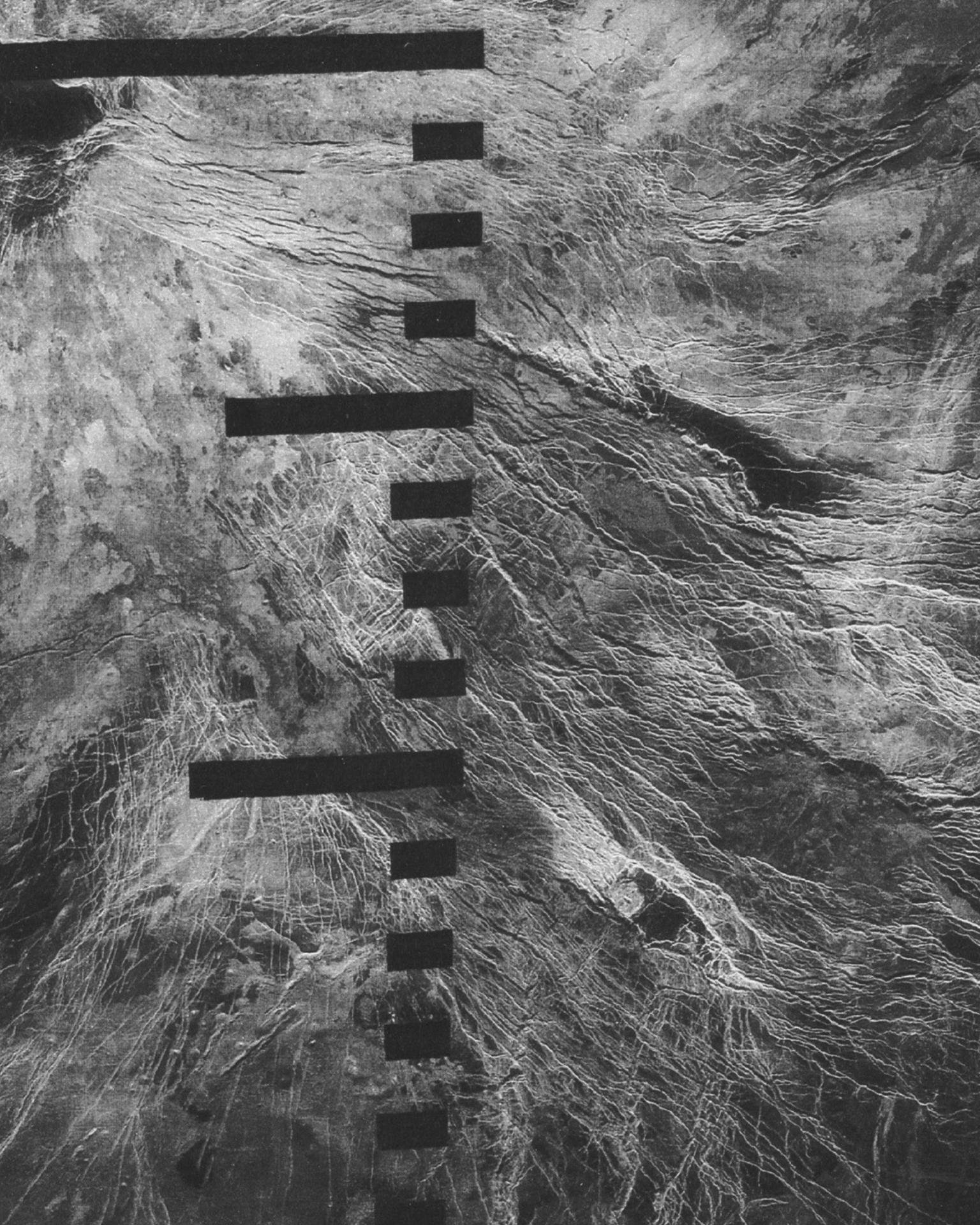14 Ibid.
15 Ibid., 143.
16 Ibid., 144.
17 Ibid.

To conclude, Leibniz does see value in Malebranche's "system of occasional causes." However, he does not believe that it is fully successful in accounting for the appearance of substance inter-action, since it only appeals to the general cause and pays no attention to the secondary causes. In order to account for the communication of substances, Leibniz's alternative view relies on his principle of preestablished harmony and the nature of substances themselves. The spontaneous, self-sufficient, and infinite nature of substances not only influences Leibniz's view of causation but also gives rise to a unique philosophy in which metaphysics provides the grounds for the study of mechanistic physics.

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